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Two models of fluid flow and mass transfer at the trailing edge of a gas slug

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Abstract—Two theoretical models for fluid flow and mass transfer at the trailing edge of a gas slug for small and large Reynolds numbers are suggested. In the case of small Reynolds numbers the creeping fluid flow at the trailing edge of a slug near a corner formed by a plane rigid wall and gas–liquid interface is investigated. The flow is caused by in-plane motion and by a fluid in the gap between a rigid wall and a gas–liquid interface. Using this model the rate of mass transfer from the bottom of a slug during gas absorption is determined. In the case of high Reynolds numbers the vortex flow at the trailing edge of the gas slug is investigated. A model of a fluid flow and mass transfer in a vortex flow in cavities is applied for the investigation of vortex formation at the trailing edge of a gas slug.

1. INTRODUCTION

Hydrodynamics of fluid flow in the region below the bottom of a gas slug was investigated in a number of studies. Campos and Guedes de Carvalho [1, 2] investigated experimentally the wakes of slugs in the range of Reynolds numbers from 25 to 1.3×10^4 using small air bubbles to trace the fluid flow. Inspection of the photographs shows that the vortex ring at the trailing edge of a gas slug forms when $Re > 50$. For $Re < 180$ the wakes are axially symmetric while in the range $180 < Re < 304$ the vortex ring in the wake oscillates. The pattern of fluid flow in the wake of a slug was also investigated by Nakoryakov *et al.* [3]. Experimental results obtained in ref. [3] confirmed the results reported in ref. [1]. Mixing induced by air slugs rising in narrow channels filled with water was investigated experimentally and theoretically by Campos and Guedes de Carvalho [2]. In the latter paper a connection was established between the hydrodynamics of the wake and the stirring effect of gas slugs. Taitel *et al.* [4] studied fluid flow in a liquid plug which was simulated by a wall jet discharging into a large reservoir. Dukler *et al.* [5] investigated the periodical distortion of a viscous boundary layer in the mixing zone at the leading edge of the liquid plug and its renewal in the bulk of a liquid plug behind the mixing region.

Mass transfer from the leading edge of the gas slug was investigated theoretically by van Heuven and Beek [6], Filla *et al.* [7] and Fominykh [8]. Van Heuven and Beek [6] investigated theoretically mass transfer from a pure gas slug during gas absorption by liquid, applying the general theory of mass transfer across free surfaces of Beek and Kramers [9]. The theoretical predictions of van Heuven and Beek [6] are in good agreement with the experimental results. Filla *et al.* [7] investigated theoretically a gas phase resistance controlled mass transfer from a single short slug during gas absorption in the presence of inert admixtures. In ref. [7], mixing in the gaseous phase was neglected and the approach by Kronig and Brink [10] for determination of a rate of mass transfer from the leading edge of a gas slug was employed. It is shown in ref. [7] that in the case of complete internal mixing, the convective mass transfer from the leading edge of a gas slug is determined by the velocity of liquid at the gas–liquid interface. The same approximation was also applied in the study by Elperin and Fominykh [11].

A comparison between the predictions of the theory based on circulating streamline flow, concentration boundary layer theory and experimental data [7] showed that for $Pe > 100$ (where $Pe = Vd/D_g$) the concentration boundary layer theory is more realistic. Mass transfer during gas absorption from a slug in the presence of inert admixtures was investigated theoretically in ref. [8] taking into account the effect of Stephan fluxes upon convective mass transfer in the gaseous phase. Absorption of a pure carbon dioxide from a single gas slug by water was investigated experimentally by van Heuven and Beek [6], Nakoryakov *et al.* [3], Niranjan *et al.* [12] and Esteves and Guedes de Carvalho [13].

Mass transfer from the cylindrical part of a gas slug can be determined from theoretical and experimental investigations of mass transfer during gas absorption to falling liquid films [14–16]. In refs. [3, 13] mass transfer from a long gas slug with a spherical head and a cylindrical body was investigated experimentally. However, to the best of our knowledge, the mass transfer at the bottom of a gas slug has not been investigated. The purpose of this investigation is to present two models for fluid flow and mass transfer at

NOMENCLATURE

a	constant in equation of turbulent diffusion, equation (76) [s^{-1}]
c	concentration of dissolved gas in liquid [$kg\ m^{-3}$]
c_1	concentration of dissolved gas in liquid surrounding vortex [$kg\ m^{-3}$]
c_2	concentration of dissolved gas in vortex [$kg\ m^{-3}$]
d	distance between walls [m]
D	coefficient of molecular diffusion in the liquid phase [$m^2\ s^{-1}$]
D_g	coefficient of molecular diffusion in gas [$m^2\ s^{-1}$]
l	diameter of a vortex [m]
Pe	Peclet number, Ud/D
Q	fluid flow rate [$m^2\ s^{-1}$]
Re	Reynolds number, Ud/ν
Sc	Schmidt number, ν/D
U	velocity of gas slug in gas–liquid slug flow [$m\ s^{-1}$]
U_1	velocity of liquid in a falling film around a slug [$m\ s^{-1}$]
U_2	velocity of liquid in a vortex on the boundary of a viscous boundary layer [$m\ s^{-1}$]
v_r, v_θ	velocity components in polar coordinates [$m\ s^{-1}$]
V	velocity of gas slug rising in a stagnant liquid [$m\ s^{-1}$]
x, y	coordinates [m].

Greek symbols

β	coefficient of mass transfer [$m\ s^{-1}$]
δ_v	thickness of viscous boundary layer [m]
θ	angle [rad]
ν	kinematic viscosity [$m^2\ s^{-1}$]
$\xi(r)$	variable in equation (35) [$m^2\ s^{-1}$]
ρ	liquid density [$kg\ m^{-3}$]
χ	constant in equation of turbulent diffusion, equation (73) [m]
ψ	stream function [$m^2\ s^{-1}$]
ω	angle velocity of liquid in the vortex [s^{-1}]
ω_δ	fluid vorticity [s^{-1}].

Subscripts

s	value at interface
∞	value at infinity
0	value at inlet.

the trailing edge of a gas slug and to estimate the contribution of the trailing edge of a gas slug to the total mass flux from a gas slug in cases of small and large Reynolds numbers.

2. CASE OF SMALL REYNOLDS NUMBERS

In the case of small Reynolds numbers ($Re < 1$) fluid flow at the trailing edge of gas slug can be considered as a creeping fluid flow in the corner formed by a free surface of the bottom of a slug and a moving wall. For simplicity we assume plane flow between two parallel walls. In a frame moving with a rising slug the wall moves down with a constant velocity U equal to the gas slug velocity in a gas–liquid flow (see Fig. 1). The flow is caused by in-plane motion of a wall and fluid flow in the gap between a wall and the bottom of a slug. The flow rate Q in a gap is determined by the distance between the walls and by the velocity of the rising slug in a stagnant fluid $Q = Vd/2$. The stream function of a creeping fluid flow at the trailing edge of the gas slug is found as a solution of a biharmonic equation [17, 18]:

$$\nabla^2(\nabla^2\psi) = 0. \tag{1}$$

The boundary conditions for equation (1) at a rigid moving wall and at the free surface at the bottom of a gas slug in the plane polar coordinates are:

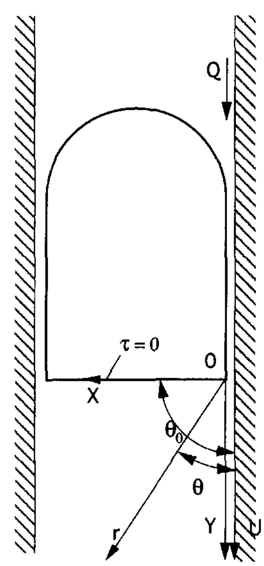

Fig. 1. Model of fluid in the tail part of a gas slug at small Reynolds numbers.

$$\frac{\partial\psi}{\partial\theta} = Ur \quad \text{and} \quad \psi = 0 \quad \text{at} \quad \theta = 0 \tag{2}$$

$$\frac{\partial^2\psi}{\partial\theta^2} = 0 \quad \text{and} \quad \psi = Q \quad \text{at} \quad \theta = \theta_0. \tag{3}$$

The components of fluid velocity can be determined from the stream function:

$$v_\theta = -\frac{\partial \psi}{\partial r} \quad v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta}. \tag{4}$$

The boundary value problem (1)–(3) can be solved by separation of variables using the method suggested in ref. [18]. Denote

$$\psi = \psi_1 + \psi_2 \tag{5}$$

where

$$\psi_1 = Urf(\theta) \tag{6}$$

and

$$\psi_2 = Qg(\theta). \tag{7}$$

Substituting equation (5) into equations (1)–(3) we transform the boundary value problem (1)–(3) into two independent boundary value problems:

$$\nabla^2(\nabla^2\psi_1) = 0 \tag{8}$$

$$\frac{\partial \psi_1}{\partial \theta} = Ur \quad \text{and} \quad \psi_1 = 0 \quad \text{at} \quad \theta = 0 \tag{9}$$

$$\frac{\partial^2 \psi_1}{\partial \theta^2} = 0 \quad \text{and} \quad \psi_1 = 0 \quad \text{at} \quad \theta = \theta_0 \tag{10}$$

and

$$\nabla^2(\nabla^2\psi_2) = 0 \tag{11}$$

$$\frac{\partial \psi_2}{\partial \theta} = 0 \quad \text{and} \quad \psi_2 = 0 \quad \text{at} \quad \theta = 0 \tag{12}$$

$$\frac{\partial^2 \psi_2}{\partial \theta^2} = 0 \quad \text{and} \quad \psi_2 = Q \quad \text{at} \quad \theta = \theta_0. \tag{13}$$

Equations (8)–(10) describe a slow viscous fluid flow in the corner formed by a free surface at the bottom of a slug and a moving rigid wall. The flow in this case is caused only by the motion of the wall. Equations (11)–(13) describe a slow viscous fluid flow in the corner formed by a free surface of the bottom of a gas slug and stagnant rigid wall. This situation occurs when a gas slug is fixed in a tube by a descending stream of liquid. Substituting $\psi_1 = Urf(\theta)$ into biharmonic equation (8) yields the ordinary differential equation for f:

$$f'''' + 2f'' + f = 0 \tag{14}$$

where primes denote differentiation with respect to θ.

The general solution (14) reads

$$f = A\sin\theta + B\cos\theta + C\theta\sin\theta + D\theta\cos\theta. \tag{15}$$

The boundary conditions for equation (14) are:

$$f = 0 \quad \text{at} \quad \theta = 0 \tag{16a}$$

$$f' = 1 \quad \text{at} \quad \theta = 0 \tag{16b}$$

$$f = 0 \quad \text{at} \quad \theta = \theta_0 \tag{17a}$$

$$f'' = 0 \quad \text{at} \quad \theta = \theta_0. \tag{17b}$$

Boundary conditions (16a) and (16b) imply that the normal fluid velocity component v_θ at a moving solid wall is equal to zero and the tangential fluid velocity component at the wall v_r is equal to the velocity of a moving solid wall. Boundary conditions (17a) and (17b) imply impermeability of a gas–liquid interface for a liquid phase and zero shear stress at a gas–liquid interface, respectively.

The constants in equation (15) are determined from the boundary conditions (16) and (17):

$$A = -\frac{2\theta_0}{\sin 2\theta_0 - 2\theta_0}$$

$$B = 0$$

$$C = \frac{2\sin^2\theta_0}{\sin 2\theta_0 - 2\theta_0}$$

$$D = \frac{\sin 2\theta_0}{\sin 2\theta_0 - 2\theta_0}. \tag{18}$$

Since $v_\theta = -Uf$, $v_r = Uf'$ the expressions for v_θ and v_r can be found explicitly:

$$v_\theta/U = -A\sin\theta - C\theta\sin\theta - D\theta\cos\theta \tag{19}$$

$$v_r/U = A\cos\theta + C(\sin\theta + \theta\cos\theta) + D(\cos\theta - \theta\sin\theta). \tag{20}$$

Substituting $\psi_2 = Qg(\theta)$ into equation (11) yields the ordinary differential equation for g:

$$4g'' + g'''' = 0. \tag{21}$$

The boundary conditions for equation (21) are:

$$g = 0 \quad \text{at} \quad \theta = 0 \tag{22a}$$

$$g' = 0 \quad \text{at} \quad \theta = 0 \tag{22b}$$

$$g = 1 \quad \text{at} \quad \theta = \theta_0 \tag{23a}$$

$$g'' = 1 \quad \text{at} \quad \theta = \theta_0. \tag{23b}$$

Boundary conditions (22a) and (22b) imply zero stream function and fluid velocity at the stationary solid wall. Boundary conditions (23a) and (23b) imply the presence of a mass source with strength Q and zero shear stresses at a gas–liquid interface, respectively.

The general solution of the differential equation (21) is:

$$g = E\sin 2\theta + F\cos 2\theta + G + H\theta. \tag{24}$$

Constants E, F, G and H in a formula (24) are determined from the boundary conditions (22) and (23):

$$E = -\frac{1}{2\theta_0 - \tan 2\theta_0}$$

$$F = \frac{\tan 2\theta_0}{2\theta_0 - \tan 2\theta_0}$$

$$G = -\frac{\tan 2\theta_0}{2\theta_0 - \tan 2\theta_0}$$

$$H = \frac{2}{2\theta_0 - \tan 2\theta_0}. \tag{25}$$

Since $v_\theta = -Q[\partial g(\theta)/\partial r]$, $v_r = (Q/r)\ [\partial g(\theta)/\partial\theta]$, the expressions for v_θ and v_r are:

$$v_\theta = 0 \tag{26}$$

$$v_r = \frac{Q}{r}(2E\cos 2\theta - 2F\sin 2\theta + H). \tag{27}$$

Using equations (15), (18)–(20) and (24)–(27) we obtain the solution of a general problem of a slow viscous fluid flow in a corner formed by the free surface at the bottom of a slug and a moving wall:

$$\psi = Ur(A\sin\theta + C\theta\sin\theta + D\theta\cos\theta) + Q(E\sin 2\theta + F\cos 2\theta + \theta H + G) \tag{28}$$

$$v_\theta/U = -A\sin\theta - C\theta\sin\theta - D\theta\cos\theta \tag{29}$$

$$v_r = U(A\cos\theta + C(\sin\theta + \theta\cos\theta) + D(\cos\theta - \theta\sin\theta)) + \frac{Q}{r}(2E\cos 2\theta - 2F\sin 2\theta + H). \tag{30}$$

In order to determine the mass transfer rate from the bottom of a gas slug we assume that the concentration boundary layer at the bottom of the slug starts to grow from the intersection line zero (see Fig. 1). Therefore the convective mass transfer is determined by fluid velocity at the bottom of a slug. Assuming that $\theta_0 = \pi/2$ we can determine fluid velocity at the bottom of the slug from equation (30):

$$v_{rs} = \frac{2U}{\pi} + \frac{4Q}{\pi r}. \tag{31}$$

Mass transfer during gas absorption at the bottom of a gas slug is governed by the equation of convective diffusion written in Cartesian coordinates x and y (see Fig. 1):

$$\left(a + \frac{b}{x}\right)\frac{\partial c}{\partial x} = D\frac{\partial^2 c}{\partial y^2} \tag{32}$$

where $a = 2U/\pi$ and $b = 4Q/\pi$.

Boundary conditions at the gas–liquid interface and at infinity are:

$$c = c_s \quad \text{at} \quad y = 0 \tag{33}$$

$$c = c_0 \quad \text{at} \quad y \to \infty. \tag{34}$$

Introducing the new variable

$$\xi = ax + b\ln x \tag{35}$$

equation (32) can be transformed into the equation similar to equation of heat conduction:

$$\frac{\partial c}{\partial \xi} = D\frac{\partial^2 c}{\partial y^2}. \tag{36}$$

The solution of equation (36) with boundary conditions (33) and (34) yields:

$$c = c_0 - (c_0 - c_s)\operatorname{erfc}(\eta) \tag{37}$$

where $\eta = y/2\sqrt{D\xi}$.

The coefficient of mass transfer at the bottom of the gas slug can be determined from the solution (37):

$$\beta = \frac{D\left(\frac{\partial c}{\partial y}\right)_{y=0}}{c_s - c_0} = \frac{D^{1/2}}{\pi^{1/2}\sqrt{(ax + b\ln x)}}. \tag{38}$$

Expression (38) implies that the coefficient of mass transfer from the trailing edge of a gas slug in the case of small Reynolds number ($Re < 1$) has the same order of magnitude as the coefficient of mass transfer from the leading edge of a gas slug [6]. Convective mass transfer from the trailing edge of a gas slug is determined by the velocity of a rising slug.

3. CASE OF LARGE REYNOLDS NUMBERS

In the case of large Reynolds numbers ($Re > 50$) a toroidal vortex is formed at the trailing edge of a gas slug (see ref. [1]). In order to study a fluid flow and mass transfer at the trailing edge of the gas slug we apply the approach suggested for investigation of fluid flow and mass transfer in cavities. For simplicity we consider a plane flow at the trailing edge of a slug is caused by a wall jet that emerges from a gap between the surface of the gas slug and the wall [4]. A schematic view of a gas slug with two vortices at the trailing edge of a gas slug is shown in Fig. 2. A detailed picture of the flow domain denoted by a dotted line in Fig. 2 is

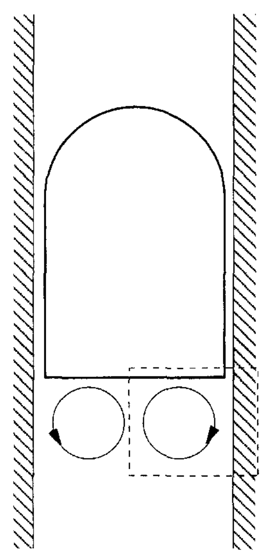

Fig. 2. General view of a vortex flow at the trailing edge of a gas slug.

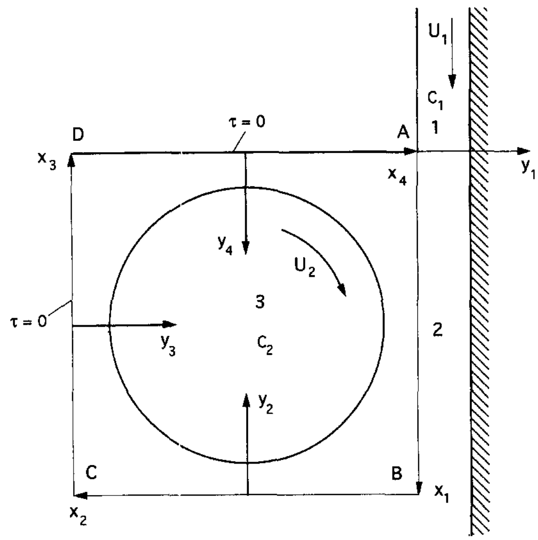

Fig. 3. Detailed picture of the flow domain denoted by a dotted line in Fig. 2.

presented in Fig. 3. Consider the case of a laminar mixing zone. Flow at the trailing edge of the gas slug can be divided into three domains: (1) region of unperturbed flow; (2) region of mixing; and (3) region of vortex flow (see Fig. 3). The region of vortex flow can be subdivided into a potential core and a viscous boundary layer [19, 20]. Velocity at the boundary of the viscous boundary layer U_2 is assumed to be constant. Fluid flow in the mixing zone AB is described by a system of equations of motion and continuity [19]:

$$u\frac{\partial u}{\partial x_1}+v\frac{\partial u}{\partial y_1}=v\frac{\partial^2 u}{\partial y_1} \tag{39}$$

$$\frac{\partial u}{\partial x_1}+\frac{\partial v}{\partial y_1}=0 \tag{40}$$

with boundary conditions

$$u=U_1 \quad v=0 \quad \text{at} \quad x_1\leqslant 0 \quad y_1\geqslant 0 \tag{41}$$

$$u=U_2 \quad v=0 \quad \text{at} \quad x_1\leqslant 0 \quad y_1\leqslant 0 \tag{42}$$

$$u=U_1 \quad \text{at} \quad x_1>0 \quad y_1\to\infty \tag{43}$$

$$u=U_2 \quad \text{at} \quad x_1>0 \quad y_1\to-\infty \tag{44}$$

where U_1 is the average fluid velocity in a falling liquid film between the gas slug–liquid interface and a tube wall. The solution of equations (39)–(40) with boundary conditions (41)–(44) yields (for details see, e.g. ref. [19]):

$$u=\frac{U_1+U_2}{2}\left(1+\frac{(U_1-U_2)}{(U_1+U_2)}\frac{2}{\sqrt{\pi}}\int_0^{\xi}\mathrm{e}^{-\tau^2}\mathrm{d}\tau\right) \tag{45}$$

where $\xi=(\sqrt{U_1})y_1/(2\sqrt{vx_1})$. The unknown velocity U_2 at the boundary of the viscous boundary layer in a vortex can be determined from an integral relation derived in ref. [19]:

$$\oint(\tau-2\mu\omega_\delta)\mathrm{d}l=0. \tag{46}$$

Integral relation (46) was derived in ref. [19] by integrating the boundary layer equations along the closed curve. The shear stress at the segment CD is equal to zero due to conditions of symmetry. The shear stress at a free surface at the bottom of the gas slug DA also equals zero. The shear stresses at the segments AB and BC are determined from equation (45):

$$\int_0^l \tau(x_1)\,\mathrm{d}x_1=\frac{\mu^{1/2}\rho^{1/2}}{\pi^{1/2}}(U_1-U_2)\sqrt{U_1 l} \tag{47}$$

$$\int_0^l \tau(x_2)\,\mathrm{d}x_2=\frac{\mu^{1/2}\rho^{1/2}}{\pi^{1/2}}U_2\sqrt{U_2 l} \tag{48}$$

where l is a vortex diameter. Equation (48) is derived under the assumption that fluid below the segment BC is stagnant. Since vorticity in the potential vortex is constant, the equation for the angular velocity of fluid in the vortex is [20]:

$$\omega=\tfrac{1}{2}\omega_\delta \tag{49}$$

therefore the expression for the velocity at the boundary of the viscous layer in the vortex is:

$$U_2=\omega l/2. \tag{50}$$

Equations (46)–(50) yield the following equation for determining U_2:

$$\left(\frac{U_2}{U_1}\right)^{3/2}+\frac{U_2}{U_1}\left(\frac{8\pi^{3/2}v^{1/2}}{U_1^{3/2}l^{1/2}}+1\right)=1. \tag{51}$$

Assuming that $(\partial u/\partial y)_{y=0}\gg(\partial u/\partial y)_{y=\delta_v}$ (see ref. [19]), expression (51) reduces to:

$$\left(\frac{U_2}{U_1}\right)^{3/2}+\frac{U_2}{U_1}=1. \tag{52}$$

Expression (52) yields $U_2/U_1=0.66$.

In the analysis of mass transfer we assume that concentration of a dissolved gas in the vortex is equal to the unknown constant value c_2 and that the concentration of dissolved gas in a fluid surrounding the vortex is equal to the concentration of the dissolved gas in a falling liquid film. The distribution of concentrations in the region of mixing AB is found from the solution of the equation of convective diffusion

$$u\frac{\partial c}{\partial x_1}+v\frac{\partial c}{\partial y_1}=D\frac{\partial^2 c}{\partial y_1^2} \tag{53}$$

with the following boundary conditions

$$c=c_1 \quad \text{at} \quad y_1\geqslant 0 \quad x_1\leqslant 0 \tag{54}$$

$$c=c_2 \quad \text{at} \quad y_1\leqslant 0 \quad x_1\leqslant 0 \tag{55}$$

$$c=c_1 \quad \text{at} \quad y_1\to\infty \quad x_1\geqslant 0 \tag{56}$$

$$c=c_2 \quad \text{at} \quad y_1\to-\infty \quad x_1\geqslant 0. \tag{57}$$

Using the solution of the hydrodynamic problem (45) we find mass and total mass flux:

$$q_c(x_1) = \frac{D(c_2-c_1)\sqrt{U_1\,Sc}}{2\sqrt{\pi \nu x_1}} \tag{58}$$

$$\int_0^l q_c(x_1)\,\mathrm{d}x_1 = \frac{D(c_2-c_1)\sqrt{U_1\,Sc\,l}}{\sqrt{\pi\nu}}. \tag{59}$$

Using the same approach we arrive at the formulas for mass flux and total mass flux at segment BC:

$$q_c(x_2) = \frac{D(c_2-c_1)\sqrt{U_2\,Sc}}{2\sqrt{\pi \nu x_2}} \tag{60}$$

$$\int_0^l q_c(x_2)\mathrm{d}x_2 = \frac{D(c_2-c_1)\sqrt{U_2\,Sc\,l}}{\sqrt{\pi\nu}}. \tag{61}$$

Mass flux $q_c(x_3)$ is equal to zero due to conditions of symmetry. Mass flux $q_c(x_4)$ is determined from the solution of the equation of convective diffusion:

$$U_2\frac{\partial c}{\partial x_4} = D\frac{\partial^2 c}{\partial y_4^2} \tag{62}$$

with the following boundary conditions

$$c = c_s \quad \text{at} \quad y_4 = 0 \tag{63}$$

$$c = c_2 \quad \text{at} \quad y_4 \to \infty \tag{64}$$

where axis y_4 is normal to x_4.

The solution of the boundary value problem (62)–(64) yields:

$$q_c(x_4) = \frac{D(c_s-c_2)\sqrt{U_2\,Sc}}{\sqrt{\pi \nu x_4}} \tag{65}$$

$$\int_0^l q_c(x_4)\mathrm{d}x_4 = \frac{2D(c_s-c_2)\sqrt{U_2\,Sc\,l}}{\sqrt{\pi\nu}}. \tag{66}$$

Since the total mass in the control volume depicted in Fig. 2 is constant

$$\oint q_c(x)\,\mathrm{d}x = 0 \tag{67}$$

therefore

$$\int_0^l q_c(x_4)\,\mathrm{d}x_4 = \int_0^l q_c(x_1)\,\mathrm{d}x_1 + \int_0^l q_c(x_2)\,\mathrm{d}x_2. \tag{68}$$

Equation (68) yields the following formula for concentration of dissolved gas in the vortex:

$$c_2 = \frac{2c_s(\sqrt{U_2})+c_1(\sqrt{U_1}+\sqrt{U_2})}{3\sqrt{U_2}+\sqrt{U_1}}. \tag{69}$$

Mass flux from the bottom of a gas slug for the case of vortex flow at the trailing edge of the slug with a laminar mixing zone is determined from formulas (65) and (69).

In the case of long slugs the coefficient of mass transfer from the cylindrical part of the gas slug is determined by the following formula:

$$\beta = \frac{2D\sqrt{U_1\,Sc}}{\sqrt{\pi\nu l}}$$

where U_1 is fluid velocity in a liquid film and l is the length of the cylindrical part of a gas slug. If the length of the cylindrical part of a slug is equal to its radius, the ratio of the coefficient of mass transfer from the trailing edge of the gas slug to the coefficient of mass transfer from the cylindrical part of a gas slug is $\sqrt{U_2/U_1} = 0.81$. For short slugs ($l_s = 1.5$–$3d$) the contribution of the bottom of a gas slug to the total mass flux from a gas slug is quite significant.

In the case of vortex formation at the trailing edge of a gas slug with turbulent mixing zone the shear stress on segments AB and BC can be obtained using the solution of the problem of mixing of two parallel streams [21, 22]:

$$\int_0^l \tau(x_1)\,\mathrm{d}x_1 = 0.0132[\sqrt{U_1(U_1-U_2)}](U_1-U_2)\rho l \tag{70}$$

and

$$\int_0^l \tau(x_2)\,\mathrm{d}x_2 = 0.0132U_2^2\rho l. \tag{71}$$

Assuming that $(\partial u/\partial y)_{y=0} \gg (\partial u/\partial y)_{y=\delta_v}$ and taking into account that equation (46) is valid in the case of turbulent mixing, we derive from equations (46), (70) and (71) the equation for U_2:

$$\left(1-\frac{U_2}{U_1}\right)^{3/2} = \left(\frac{U_2}{U_1}\right)^2. \tag{72}$$

Equation (72) implies that $U_2/U_1 = 0.55$ in the case of vortex flow with a turbulent mixing zone. Then the distribution of concentration of the dissolved gas in the mixing zone is found from the solution of the equation of turbulent diffusion [22]:

$$u\frac{\partial c}{\partial x_1} + v\frac{\partial c}{\partial y_1} = \chi(U_1-U_2)\frac{\partial^2 c}{\partial y_1^2} \tag{73}$$

with boundary conditions (54)–(57). From the solutions of equations (73) and (54)–(57) we obtain:

$$\int_0^l q_c(x_1)\,\mathrm{d}x_1 = 0.05l[\sqrt{U_1(U_1-U_2)}](c_2-c_1) \tag{74}$$

$$\int_0^l q_c(x_2)\,\mathrm{d}x_2 = 0.05lU_1(c_2-c_1). \tag{75}$$

Mass flux from the bottom of the gas slug is determined from the solution of the equation of turbulent diffusion:

$$U_2 \frac{\partial c}{\partial x_4} = \frac{\partial}{\partial y_4}\left((D+ay_4^2)\frac{\partial c}{\partial y_4}\right) \quad (76)$$

with boundary conditions (63) and (64) (see ref. [14]):

$$q_c(x_4) = \beta(x_4)(c_s - c_2) = \frac{D^{1/2}U_2^{1/2}}{\pi^{1/2}x_4^{1/2}} \times \left(1+\frac{ax_4}{U_2}-\frac{ax_4^2}{8U_2^2}\right)(c_s-c_2) \quad (77)$$

$$\int_0^l q_c(x_4)\,\mathrm{d}x_4 = (c_s-c_2)\int_0^l \beta(x_4)\,\mathrm{d}x_4 = (c_s-c_2)\frac{2D^{1/2}U_2^{1/2}l^{1/2}}{\pi^{1/2}}\left(1+\frac{al}{3U_2}-\frac{a^2l^2}{40U_2^2}\right). \quad (78)$$

Equations (67), (74), (75) and (78) yield the following formula for the concentration of dissolved gas in the vortex:

$$c_2 = \frac{c_s\int_0^l \beta(x_4)\,\mathrm{d}x_4 + c_1 0.05l([\sqrt{U_1(U_1-U_2)}]+U_2)}{\int_0^l \beta(x_4)\,\mathrm{d}x_4 + 0.05([\sqrt{U_1(U_1-U_2)}]+U_2)}. \quad (79)$$

Mass flux from the bottom of the gas slug for the case of vortex formation at the trailing edge of the slug with a turbulent mixing region is determined by formulas (77) and (79).

4. CONCLUSIONS

In this study we analyzed fluid flow and mass transfer at the trailing edge of a gas slug for small and large Reynolds numbers. It was shown that in the case of small Reynolds numbers fluid flow at the trailing edge of the gas slug can be described as a creeping viscous fluid flow in the corner between the free surface at the bottom of the gas slug and the moving wall. In the case of large Reynolds numbers fluid flow below the bottom of the gas slug can be described as a vortex flow with laminar and turbulent mixing zones.

The derived formulas (38), (65), (69), (77) and (79) show that the coefficient of mass transfer from the trailing edge of a gas slug is of the same order of magnitude as the coefficient of mass transfer at the leading edge of a gas slug and the coefficient of mass transfer at the cylindrical part of a gas slug. In the case of short slugs ($l_s = 1.5$–$3d$) the contribution of the bottom part of a gas slug to the total mass flux from a gas slug is quite large. In the case of long slugs ($l_s \geqslant 10d$) the contribution of the bottom of a gas slug to the total mass flux from a gas slug is negligibly small.

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